Histograms

Thanks to David Jacobs for the use of some slides

Grayscale Images

- Matrix of scalars
- Usually 8 bit: 0 – 255
  - 0 is black, 255 is white

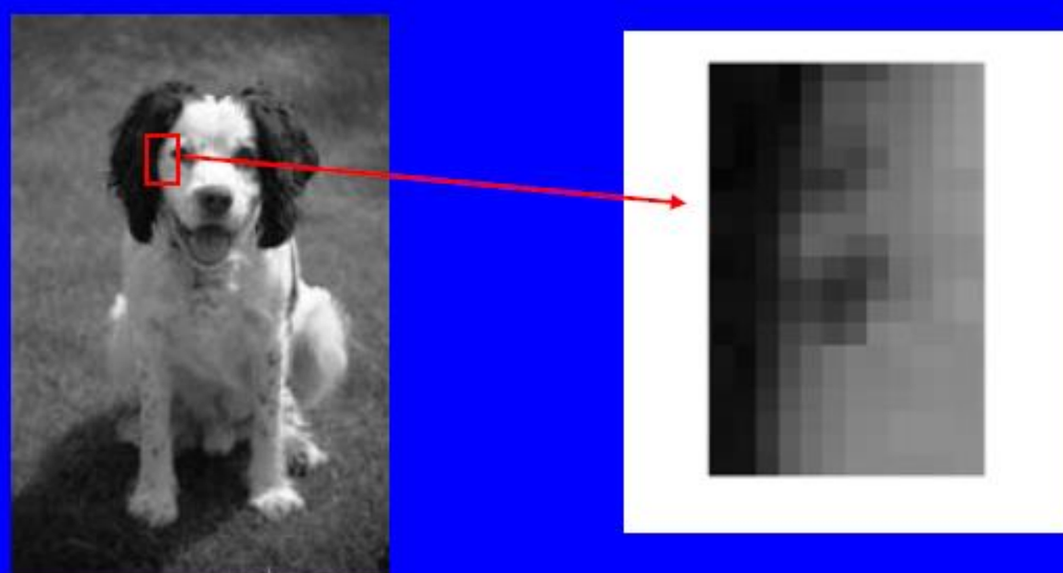


Image Processing

Image Processing means transforming images into new images

Simplest image processing treats every pixel independently.

I is input image, J is output image.

I(x,y), J(x,y) are corresponding pixels.

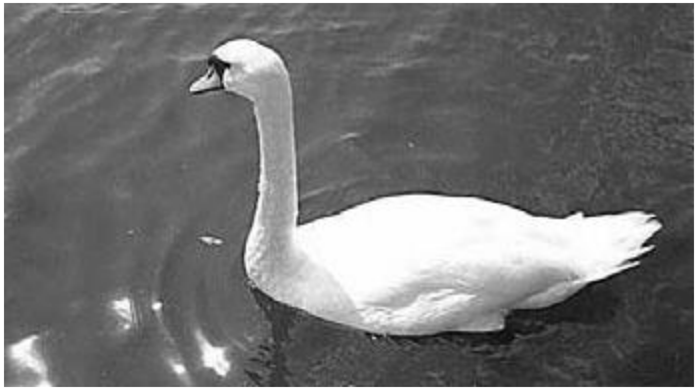
$J(x,y) = f(I(x,y))$.

Thresholding

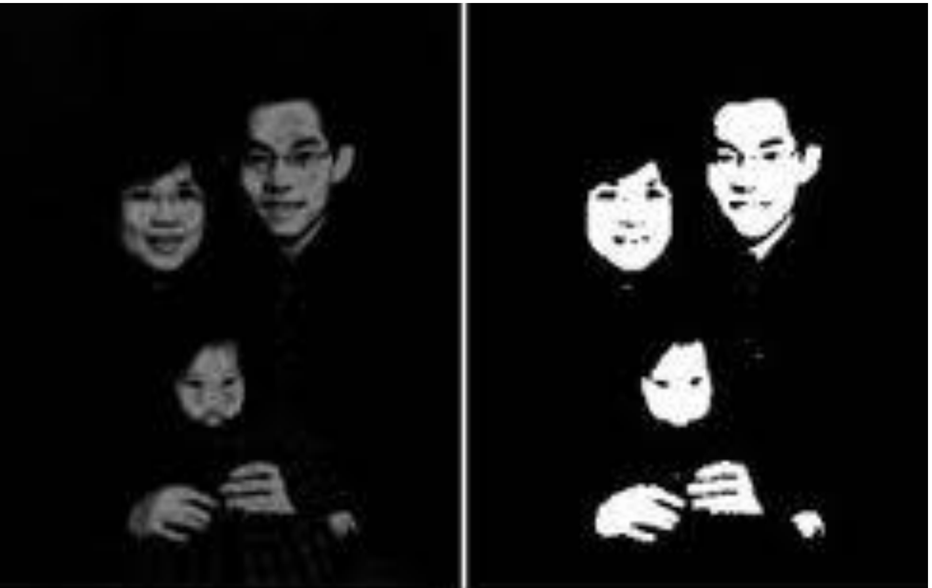
One of the simplest operations we can perform on an image is *thresholding.*

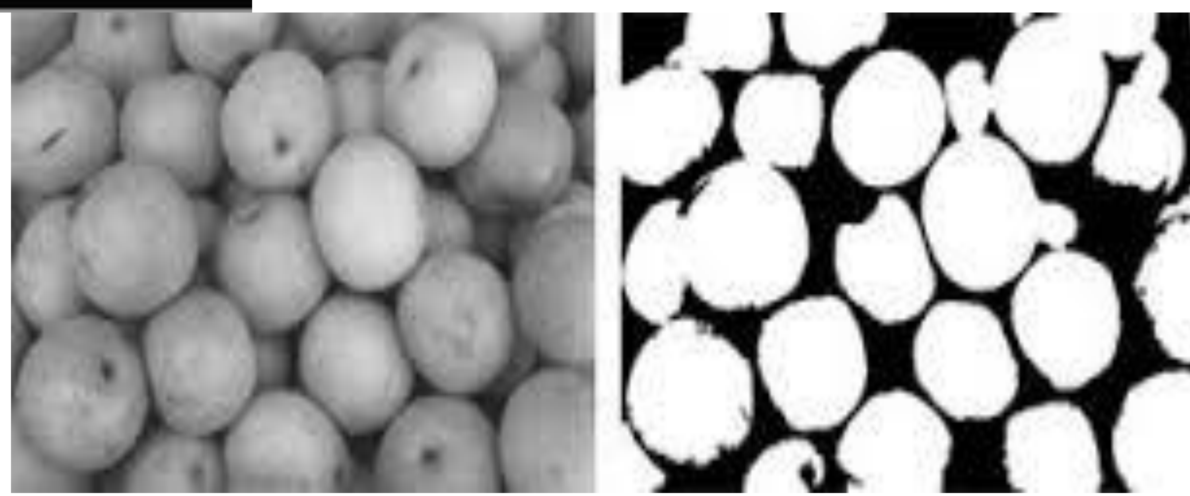
For example, if we take the swan image
and threshold it with a threshold of T,
we make all pixels >= T into 1, and all
pixels < T into 0.

Threshold T=128

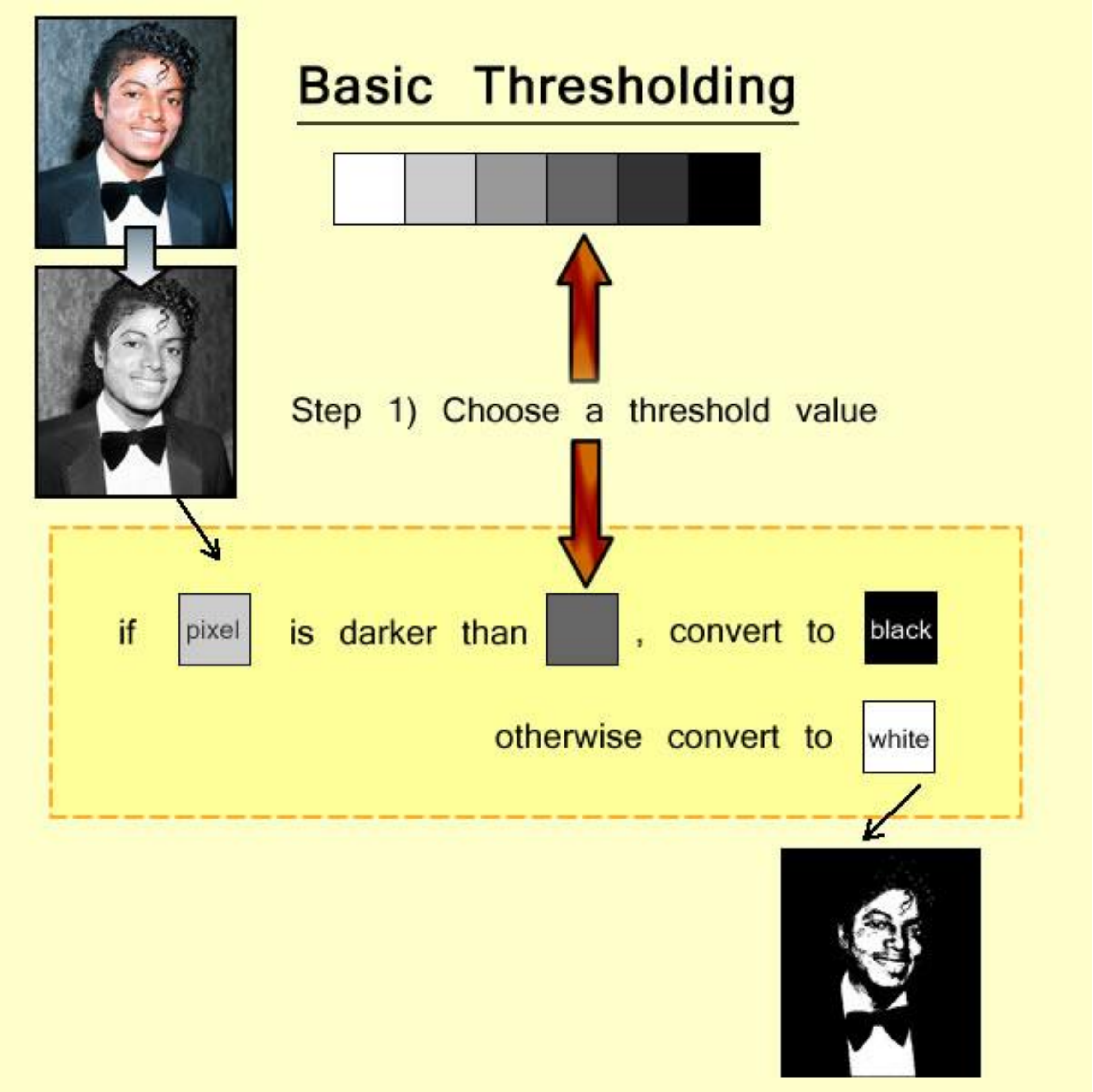


Examples





Summary

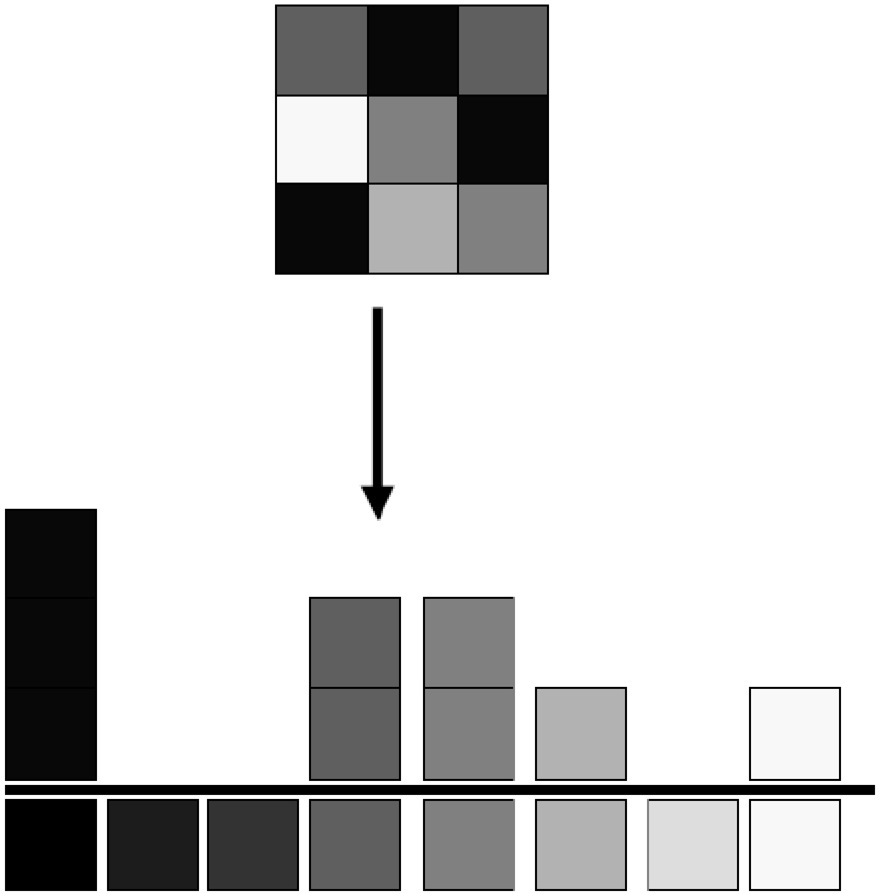


Basic Thresholding
Step 1) Choose a threshold value
if
pixel
is darker than
, convert to
black
otherwise convert to
white

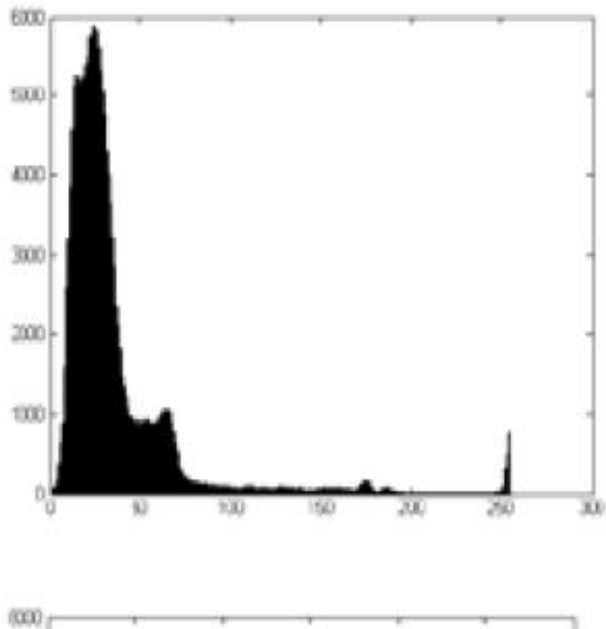
Grayscale Histogram

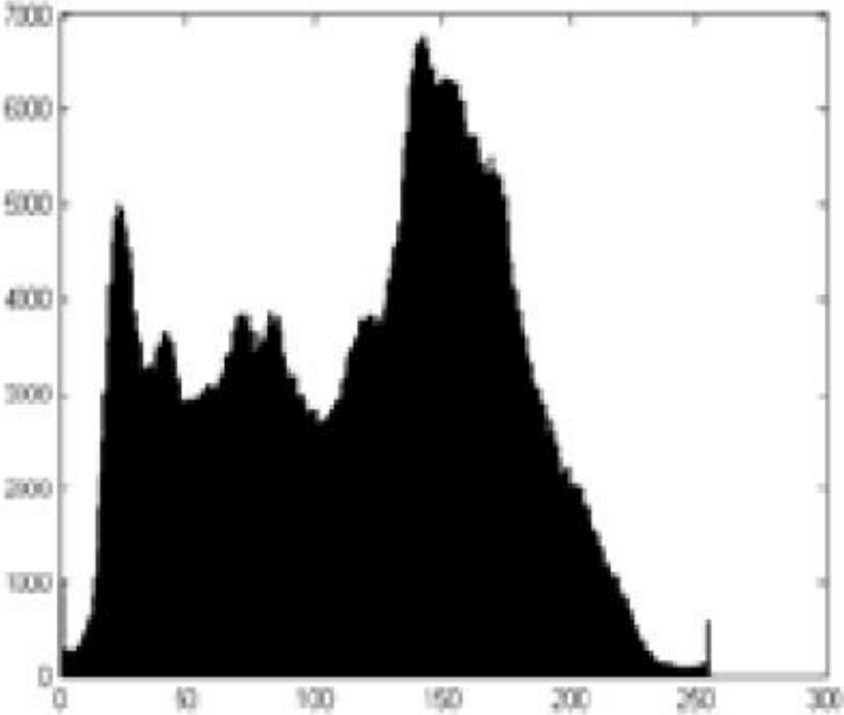
- Count intensities
- Normalize
- What determines Histogram?
- – Contrast, aperture, lighting levels, scene,

A simple image and its histogram



Examples




7000
6000
5000
4000
3000
2000
1000
0
0
50
100
150
200
250
300

Definition of histogram

- To write this down, we might say that we have an image, I, in which the intensity at pixel with coordinates (x,y) is I(x,y). We would write the histogram h, as h(i) indicating that intensity i, appears h(i) times in the image. If we let the expression (a=b) have the value 1 when a=b, and 0 otherwise, we can write for histogram h(i):

$$h(i) = \sum_x \sum_y I(x,y) = i$$

Histograms allow image manipulation

- One reason to compute a histogram is that it allows us to manipulate an image by changing its histogram. We do this by creating a new image, J, in which:

$$J(x,y) = f(I(x,y))$$

- The trick is to choose an f that will generate a nice or useful image. Typically, we choose f to be monotonic. This means that: if $u<v$ then $f(u) < f(v)$. Non-monotonic functions tend to make an image look truly different, while monotonic changes will be more subtle.

Histogram Equalization

- The idea is to spread out the histogram so that it makes full use of the dynamic range of the image.
- For example, if an image is very dark, most of the intensities might lie in the range 0-50. By choosing f to spread out the intensity values, we can make fuller use of the available intensities, and make darker parts of an image easier to understand.
- If we choose f to make the histogram of the new image, J, as uniform as possible, we call this **histogram equalization.**

How to do it

- **Cumulative Distribution Function (CDF).**

This encodes the fraction of pixels with an intensity that is equal to or less than a specific value.

If h is a histogram and C is a CDF, then h(i) indicates the number of pixels with intensity of i, while ,

$$C(i) = \sum_{j<=i} h(j)/N$$

indicates the fraction of pixels with intensity less than or equal to i, assuming the image has N pixels.

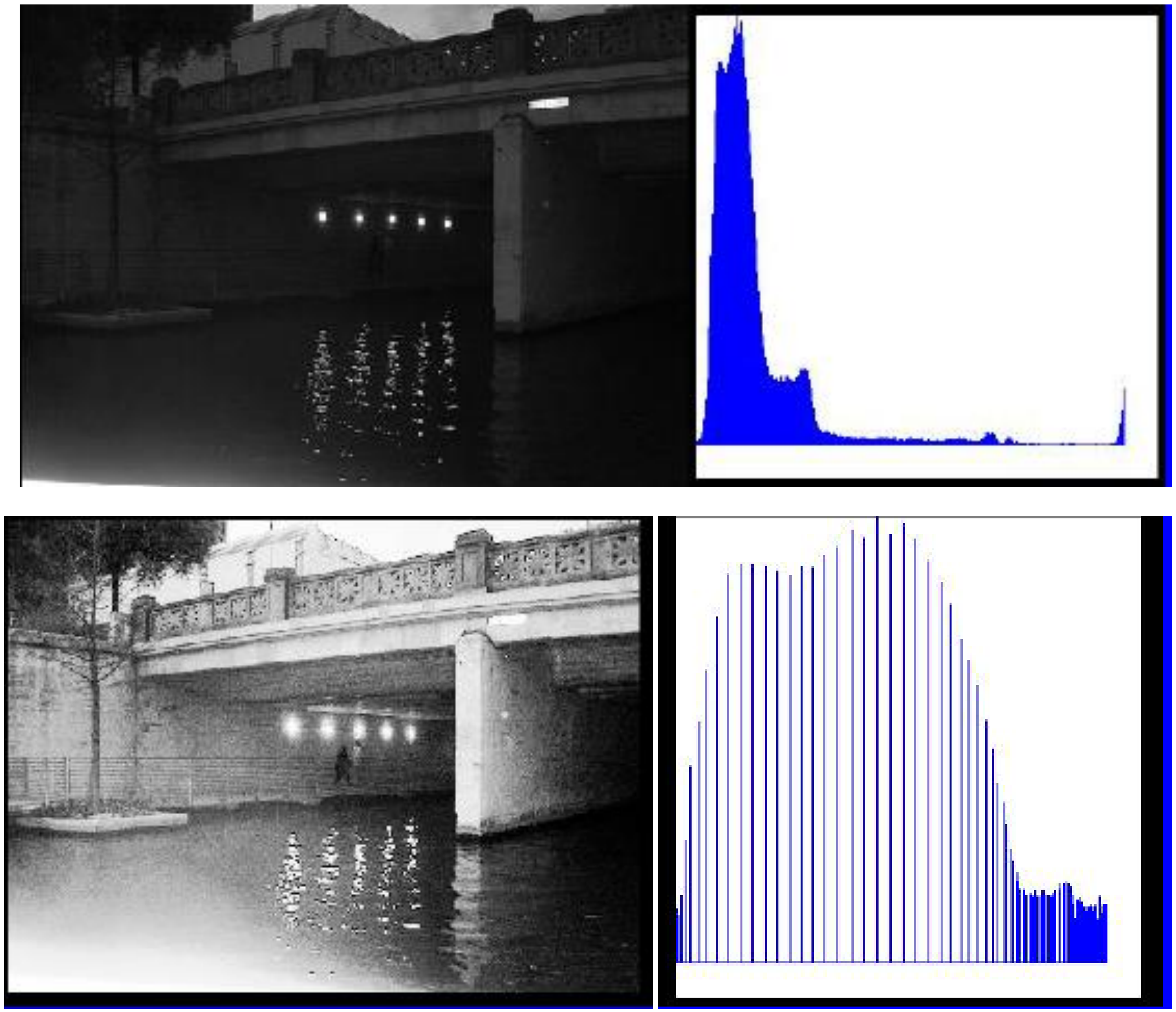
Actual construction

- g is the histogram of J, and D its CDF.

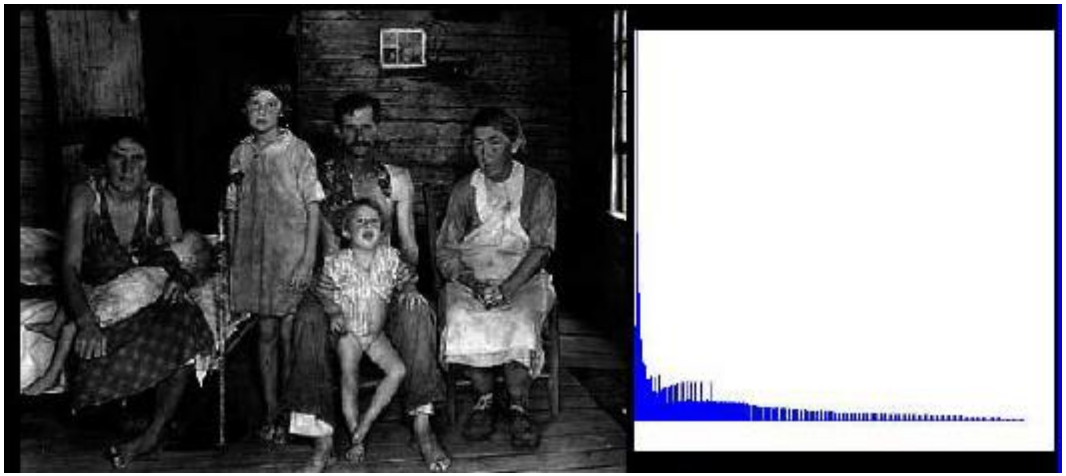
- If there are k intensity levels in an image, then we want g=(N/k, N/k, …), and D=(1/k, 2/k,3/k, …). This means that we want D(i) = i/k. Notice that C(i) = D(f(i)). That is, all the pixels in I that have an intensity less than or equal to i will have an intensity less than or equal to f(i) in J (since f is monotonic, if $j<i$, $f(j) < f(i)$).

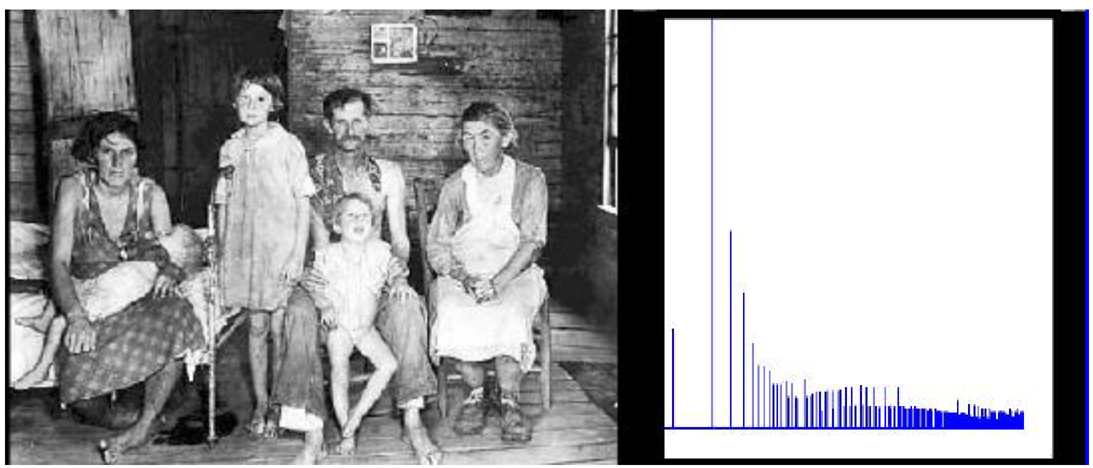
- Putting these together, we have D(f(i))=f(i)/k=C(i), so f(i) = kC(i).

Examples of histogram equalization



Examples





Comparing histograms

- SSD

Let h and g be two histograms.

$$\|\mathrm{h} - \mathrm{g}\| = \sum_{i=1}^{N} \left(h(i) - g(i) \right)^{2}$$

- Cosine

$$\cos(h, g) = \frac{\langle h, g \rangle}{\|h\| \|g\|}$$

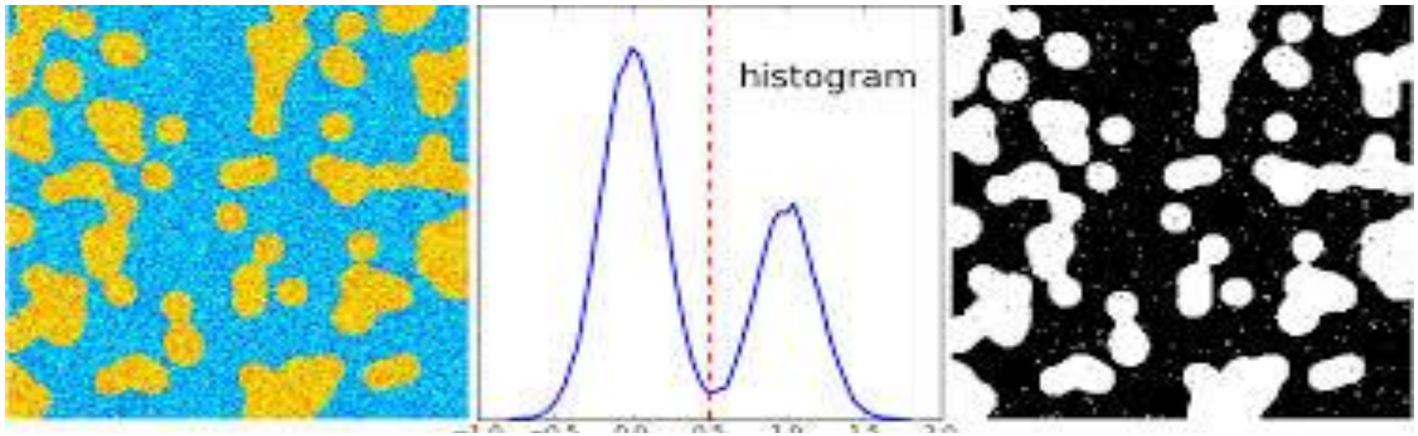
Treating histograms as probability distributions

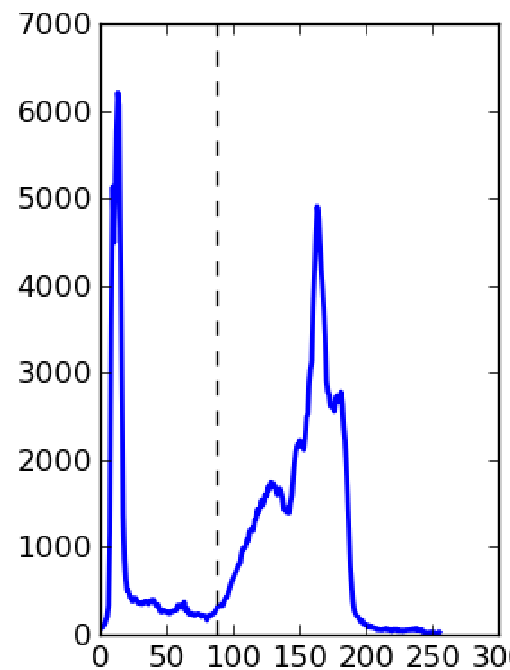
- Chi-Squared

$$\chi^2(h_i, h_j) = \frac{1}{2}\sum_{m=1}^{K}\frac{[h_i(m) - h_j(m)]^2}{h_i(m) + h_j(m)}$$

- Smoothing probability distributions
  - Use bigger buckets.
  - Add a constant value (eg., 1) to every bucket.
  - Gaussian smoothing

Other uses of histograms





Mammograms